A Note on Optimal Taxation, Status Consumption, and Unemployment*

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ABSTRACT

Existing research on optimal taxation in economies with status-driven relative consumption assumes that the labor market is competitive, despite the fact that real world labor markets are typically characterized by involuntary unemployment. We show how the marginal tax policy ought to be modified to simultaneously account for positional consumption externalities and equilibrium unemployment, and find that interaction effects between these two market failures are crucial determinants of the marginal tax structure. In certain cases, the policy incentive to tax away positional externalities vanishes completely, and negative positional externalities may even lead to lower marginal taxation, under involuntary unemployment.

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1. Introduction

A voluminous literature based on happiness research and questionnaire-experiments shows that people are concerned with their relative consumption or relative income compared to referent others,[1] which is often interpreted as a desire for social status manifested through conspicuous consumption.[2] In fact, a large fraction of the utility gain to the individual of increased consumption seems to be driven by concerns for relative consumption, suggesting, in turn, that the welfare cost of the associated positional externalities may be considerable. [3] Consequently, a literature dealing with tax policy implications of relative consumption concerns shows that these externalities are likely to motivate much higher marginal tax rates than implied by conventional models of optimal taxation (in which people are assumed to be completely non-positional).[4]

However, earlier studies dealing with tax policy implications of relative consumption concerns are based on model-economies where the labor market is competitive and thus characterized by full employment. This assumption is clearly at odds with reality, since involuntary unemployment is a serious social problem in many countries. In the present paper, we integrate involuntary unemployment in a model of optimal taxation and status consumption. This extension is important for at least two reasons. First, and foremost, the tax policy implications of relative consumption concerns are shown to largely depend on whether the economy is characterized by full employment or unemployment in equilibrium. Therefore, since positional externalities are prevalent alongside equilibrium unemployment, their policy implications ought to be addressed simultaneously. Second, the presence of equilibrium unemployment has itself important implications for the marginal tax structure, and implies that an employment-related motive for taxation typically works to push up the marginal income tax rate.[5] An interesting question is, therefore, whether relative consumption concerns just strengthen this

[1] See, e.g., Easterlin (2001), Johansson-Stenman et al. (2002), Blanchflower and Oswald (2005), Ferrer-i-Carbonell (2005), Luttmer (2005), Solnick and Hemenway (2005), and Carlsson et al. (2007).

[2] Veblen (1899) is the standard reference for conspicuous consumption, although some implications thereof were touched upon already by earlier authors, while Duesenberry (1949) was the first to develop a theory of consumption behavior in this context.

[3] See Wendner and Goulder (2008) for an overview, and Frank (1985, 2005) for illuminating discussions of positional externalities and their policy implications.

[4] See, e.g., Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Dupor and Liu (2000), Aronsson and Johansson-Stenman (2008, 2010, 2018), and Kanbur and Tuomala (2013).

[5] See, e.g., Marceau and Boadway (1994), Aronsson and Sjögren (2004), and Aronsson and Micheletto (forthcoming).

motive for marginal taxation in economies with equilibrium unemployment, or whether there are counteracting mechanisms as well from combining these two market failures.

We consider a model of optimal income taxation; here extended by a minimum wage policy leading to involuntary unemployment among the low-skilled.[6] Section 2 describes the model, and the main result is presented in Section 3. The results show that positional externalities and unemployment interact in the marginal tax structure in ways which were not predictable from model-economies where each of these phenomena is examined in isolation. Under full employment, our model would imply that the positional externality contributes to higher marginal taxation for everybody, as in the existing literature, where the mark-up is determined by the average degree of positionality. However, if the equilibrium is characterized by unemployment among the low-skilled, and if the number of workers and the hours of work per worker are substitutes in production, as is often assumed, then positional externalities typically lead to a lower marginal income tax for low-skilled workers, whereas the marginal income tax implemented for the high-skilled increases more in response to positional externalities than under competition in the labor market. This means that the policy response is asymmetric and works in the direction of a more progressive marginal tax structure, even if the underlying positional externality is atmospheric.

The intuition is that decreased labor supply per low-skilled worker contributes to the positional consumption externality via two channels: First, it reduces their consumption and thus also the corresponding positional externality. This mechanism is the same as without unemployment and implies that the optimal policy response is to increase the marginal income tax rate. Second, a decrease in the labor supply per low-skilled worker also reduces the unemployment, such that more people will contribute to the externality through their choices of work hours and consumption. This mechanism instead works to decrease the optimal marginal income tax rate. In a first-best environment where earnings ability is observable, these two effects cancel out, meaning that the corrective tax element vanishes completely, while the latter effect dominates the former in a second-best setting where earnings ability is private information.

[6] The rationale for focusing on a case where only the low-skilled are subject to unemployment is, of course, that unemployment is typically much more frequent and persistent among the low-skilled than among the high-skilled. Marceau and Boadway (1994) use a model similar to ours for analyzing the implications of a minimum wage policy targeting the low-skilled. Similarly, Aronsson and Micheletto (forthcoming) use a model of optimal income taxation for examining the tax policy implications of efficiency wage formation among the low-skilled. However, none of these studies integrates status consumption in the analysis, which is the main issue here.

For high-skilled individuals, who by assumption do not run the risk of becoming unemployed, a decrease in the hours of work does not only reduce their own contribution to the positional externality, it also reduces the contribution to this externality by the low-skilled through complementarities in the production function. The reason is that decreased labor supply among the high-skilled will increase the unemployment among the low-skilled, and thus also decrease the number of persons that can contribute to the positional externality through their labor supply behavior. Overall, the assumption of competition in the labor market invoked in earlier comparable studies on optimal taxation is, therefore, not innocuous. Section 4 summarizes the main findings, while proofs and mathematical results supporting the analysis are available in the Appendix.

2. The Model

Consider an economy with *N* individuals, among which N^1 have low earnings-ability (type 1) and N^2 have high earnings-ability (type 2). This means that the high-ability type earns a higher before-tax wage rate than the low-ability type. The production sector comprises identical, competitive firms producing a homogenous good. We start by describing the production side of the economy and then continue with the consumers.

2.1 Production

We assume that the production sector is characterized by identical, competitive firms, and we abstract from the entry of new firms by assuming that their number is fixed. Since the exact number of firms is not important for the results, as long as they behave competitively, we normalize their number to one for notational convenience. Let $N_e^1 \leq N^1$ denote the number of employed persons of the low-ability type. The production function can then be written as $F(L^1, L^1) = F(l^1 N_e^1, l^2 N^2)$, where l^i denotes the hours of work per employee of ability-type *i*, and L^i denotes the total number of work hours by ability-type *i* in the economy as a whole (the hours of work per employee times the number of employed persons). We assume that the production function is increasing and strictly concave in each argument, i.e., $F_{L^1} > 0$, $F_{L^2} > 0$, $F_{L^1L^1} < 0$, and $F_{L^2L^2} < 0$, and that the production factors are weak technical complements such that $F_{L^1L^2} \geq 0$, where a single subscript denotes first-order partial derivate and double subscript second-order partial derivative. If $F_{L^1L^2} > 0$, an increase in the high-skilled individuals' labor supply will reduce the unemployment among the low-skilled (as explained below); this influence is absent if $F_{L^1L^2} = 0$.

Firms decide on the number of individuals of each type to employ, while treating the hours of work per employee as exogenous.[7] Following Marceau and Boadway (1994), we assume that the labor market facing high-ability individuals is competitive, meaning that the hourly wage rate, w^2, adjusts in such a way that all of them are employed in equilibrium, while the labor market facing low-ability individuals is characterized by a minimum-wage , $w^1 = \bar{w}^1$, which exceeds the market clearing wage rate.[8] The firm thus satisfies the following first-order conditions:

$$F_{L^1}(l^1N_e^1, l^2N^2) = \bar{w}^1 \tag{1a}$$

$$F_{L^2}(l^1N_e^1, l^2N^2) = w^2 . \tag{1b}$$

As such, since $\bar{w}^1$ is fixed, N_e^1 and w^2 will adjust to preserve the first-order conditions. This opens up for equilibrium unemployment, where $N^1 - N_e^1$ low-ability individuals are involuntarily unemployed. In our model, where all low-ability individuals are identical ex-ante by assumption, the employment status can be thought of as the outcome of a random draw.[9]

The following comparative statics derived from equation-system (1a)-(1b) will be used below:

$$\frac{\partial N_e^1}{\partial l^1} = -\frac{N_e^1}{l^1} < 0 \tag{2a}$$

$$\frac{\partial N_e^1}{\partial l^2} = -\frac{F_{L^1L^2}}{F_{L^1L^1}}\frac{N^2}{l^1} \geq 0 . \tag{2b}$$

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[7] As long as the individuals decide on their labor supply, and the government has access to a general income tax by which to control the underlying incentives, this assumption is natural and accords well with many earlier studies on optimal nonlinear income taxation. An alternative assumption would be that each firm can directly affect the hours of work per employee in some way in addition to deciding on the number of persons to employ. In that case, a (quire realistic) hiring cost would induce firms to respond to the minimum wage policy by employing fewer low-ability individuals instead of reducing the hours of work per employee.

[8] The crucial aspect here is that the minimum wage is high enough to cause unemployment in equilibrium. It is not important for the qualitative results derived below whether this minimum wage is decided on by the government or by an economy-wide trade union (which acts as a Nash-competitor vis-à-vis the government). In the Appendix, we exemplify by presenting the first-order condition for a minimum wage decided on by the government.

[9] An alternative would be to assume that individuals differ in terms of labor market seniority, as in Oswald (1993), such that the least senior worker is the first to become unemployed, and so on. This change of assumption would not affect the results derived below.

2.2 Consumer Preferences and Choices

Following Aronsson and Johansson-Stenman (2008, 2018), we assume that each consumer derives utility from her own consumption and leisure time, respectively, as well as from her relative consumption compared to referent others. The preferences are characterized by the following (possibly type-specific) utility function:

$$U^i = v^i(c^i, z^i, c^i - \bar{c}) = u^i(c^i, z^i, \bar{c}) \text{ for } i=1,2. \tag{3}$$

The utility facing an unemployed individual (which is of type 1 by assumption) will be written as

$$U^u = v^u(c^u, 1, c^u - \bar{c}) = u^u(c^u, 1, \bar{c}). \tag{4}$$

In equations (3) and (4), c denotes the individual's (absolute) consumption, $c - \bar{c}$ denotes the individual's relative consumption compared to a measure of reference consumption, $\bar{c}$, and $z = 1 - l$ denotes the time spent on leisure if employed (with the time-endowment normalized to unity); if unemployed, all time is spent on leisure. Despite that unemployment allows the individual to spend more time on leisure, several earlier studies show that unemployment leads to lost well-being beyond the income loss it causes.[10] As such, a switch from employment to unemployment may imply a drop in the utility level in addition to the net effect on well-being caused by the associated changes in consumption and leisure. The distinction between the functions $v^1(\cdot)$ and $v^u(\cdot)$ serves to capture this possibility. The function $u^i(\cdot)$ is a convenient reduced form to be used in some of the calculations below. Finally, we follow much earlier research in assuming that the reference consumption, $\bar{c}$, is given by the average consumption in the economy as a whole[11]

$$\bar{c} = \frac{N_e^1 c^1 + (N^1 - N_e^1)c^u + N^2 c^2}{N}. \tag{5}$$

Note that $\bar{c}$ depends directly on the consumption choices made by all individuals and indirectly on their hours of work via N_e^1.

Let $T(w^i l^i)$ denote the income tax (positive or negative) paid by an employed individual of ability-type *i*. The budget constraint facing this individual can then be written as

$$w^i l^i - T(w^i l^i) = c^i. \tag{6}$$

[10] See, e.g., Clark and Oswald (1994) and Winkelmann and Winkelmann (1998).

[11] This is the conventional assumption in earlier studies on optimal taxation and relative consumption and implies that each individual compares her own consumption with the economy-wide average. Some other comparison forms (other than mean-value comparisons) are examined by Aronsson and Johansson-Stenman (2010, 2018).

Each individual treats the before-tax wage rate, the parameters of the tax system (including the structure of marginal taxation), the level of public expenditure, and the reference consumption as exogenous. The first-order condition for work hours becomes

$$MRS^i_{z,c} \equiv \frac{u^i_z}{u^i_c} = \frac{v^i_z}{v^i_c + v^i_\Delta} = w^i\left(1 - T'(w^i l^i)\right) \quad (7)$$

where $T'(w^i l^i)$ is the marginal tax rate, and $v^i_\Delta = \partial v^i / \partial(c^i - \bar{c})$ measures the marginal utility of relative consumption. As such, $u^i_c = v^i_c + v^i_\Delta$ denotes the individual's total marginal utility of consumption. Unemployed individuals make no decisions in the model; instead, if the individual is unemployed, the consumption is given by $c^u = b$, where b denotes an unemployment benefit.

The tax policy implications of relative consumption concerns depend on the extent to which these concerns affect individual well-being. It is, therefore, useful to measure these concerns through indicators developed in empirical literature on relative consumption and well-being. Following Johansson-Stenman et al. (2002), the strength of the preference for relative consumption is captured by the degree of positionality

$$\alpha^j = \frac{v^j_\Delta}{v^j_c + v^j_\Delta}$$

for j=1, 2, u, i.e., the share of the utility gain of increased consumption that is attributable to increased relative consumption. The average degree of positionality then becomes

$$\bar{\alpha} = \frac{N^1_e \alpha^1 + (N^1 - N^1_e)\alpha^u + N^2 \alpha^2}{N}. \quad (8)$$

Since the relative consumption concerns are driven by mean-value comparisons in our model, we can interpret $\bar{\alpha}$ as measuring the marginal positional externality per unit of consumption. In their literature review, Wendner and Goulder (2008) conclude that the degree of positionality is around 0.2-0.4 on average, while Carlsson et al. (2007) find that this degree is around 0.5 for income (which reflects the possibility to consume in general) and even higher for certain visible goods such as cars and homes.

3. The Optimal Tax Problem and Results

We assume that the policy maker is maximizing a utilitarian social welfare function.[12] Following much earlier literature, the social decision-problem will be written as a direct decision-problem (i.e., in terms of quantities instead of tax parameters) as follows:

$$\underset{c^1,l^1,c^2,l^2,b}{Max} \; N_e^1 v^1(c^1, z^1, c^1 - \bar{c}) + N^2 v^2(c^2, z^2, c^2 - \bar{c}) + \left(N^1 - N_e^1\right) v^u(b, 1, b - \bar{c})$$

subject to a resource constraint for the economy as a whole and a self-selection constraint. Furthermore, the policy maker knows how the measure of reference consumption (equation [5]) is determined and attempts to internalize the positional externality, as well as recognizes that the number of employed low-ability individuals is determined through the labor demand function, $N_e^1 = N_e^1(l^1, l^2, \bar{w}^1)$.

The resource constraint can be written as[13]

$$F(N_e^1 l^1, N^2 l^2) = N_e^1 c^1 + (N^1 - N_e^1) b + N^2 c^2 . \tag{9}$$

We assume that the policy maker can observe (and thus tax) earnings, whereas individual earnings-ability is private information, and that the policy maker wants to redistribute from high-ability individuals to (employed and unemployed) low-ability individuals. By assuming that U^2 always exceeds the utility that a high-ability individual would experience if being unemployed, which appears to us as a realistic assumption, we only need to prevent high-ability individuals from mimicking employed low-ability individuals, which is accomplished via the following self-selection constraint:[14]

$$U^2 = v^2(c^2, z^2, c^2 - \bar{c}) \geq v^2(c^1, 1 - \phi l^1, c^1 - \bar{c}) = \hat{U}^2 . \tag{10}$$

The left hand side of the weak inequality (10) is the utility of the high-ability type, whereas the right hand side is the utility facing a potential mimicker (a high-ability individual mimicking the income of the low-ability type). $\phi = w^1 / w^2 = \bar{w}^1 / w^2 < 1$ is the relative wage rate and $\phi l^1 < l^1$ the potential mimicker's labor supply. Equation system (1a)-(1b) implies $\partial \phi / \partial l^1 = 0$ and $\partial \phi / \partial l^2 > 0$.

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[12] The utilitarian social welfare function simplifies the analysis; it is not essential for the results presented below. A general social welfare function, in which identical individuals are given the same welfare weight, would give the same policy rules for marginal taxation as those presented below.

[13] Pure profits (if any) are taxed away completely.

[14] A similar approach to describing the potential mimicker was taken by Marceau and Boadway (1994) and Aronsson and Micheletto (forthcoming). In the latter study, which distinguishes between two types of jobs, a potential mimicker can either mimic the income or the job of the low-ability type (depending on which option is preferable). Such an extension of the model would not affect the qualitative results derived below.

3.1 Some Preliminaries

Let λ denote the Lagrange multiplier of the self-selection constraint and γ the Lagrange multiplier of the resource constraint (interpretable as the marginal cost of public funds measured in utility units). To simplify the notation, and provide a starting point for the analyses to follow, it is instructive to begin by presenting the marginal tax policy corresponding to the special case where (1) the consumers are completely non-positional (such that $\alpha^1 = \alpha^2 = \alpha^u \equiv 0$), and (2) there is full employment among the low-skilled (i.e., $N_e^1 \equiv N^1$). Consider Observation 1, in which we have used the short notation $\hat{u}^2 = u^2(c^1, 1-\phi l^1, \bar{c})$.[15]

Observation 1. (Stiglitz, 1982) *If* $\alpha^j \equiv 0$ *for j=1, 2, u, and* $N_e^1 \equiv N^1$ *the policy rules for marginal income taxation take the following conventional form:*

$$\tau^1 = \lambda^{1,*}\left(\frac{u_z^1}{u_c^1} - \phi\frac{\hat{u}_z^2}{\hat{u}_c^2}\right) \qquad (11a)$$

$$\tau^2 = -\lambda^{2,*}\frac{\partial\phi}{\partial l^2} l^1 \qquad (11b)$$

where $\lambda^{1,*} = \dfrac{\lambda \hat{u}_c^2}{\gamma N_e^1 w^1} > 0$ *and* $\lambda^{2,*} = \dfrac{\lambda \hat{u}_z^2}{\gamma N^2 w^2} / > 0$.

Equations (11a) and (11b) are analogous to the policy rules for marginal income taxation derived by Stiglitz (1982), who examined an economy with full employment and non-positional consumers, and will therefore not be given any further interpretation here. Just note that $\tau^1 > 0$ if all individuals share a common utility function, and $\tau^2 < 0$.

Before turning to the main results, two additional preliminary results are useful, by showing how relative consumption concerns and unemployment, if introduced separately, would modify the marginal tax policy described in Observation 1. In Observations 2 and 3, we characterize the implications of each such extension in turn. In doing so, and to simplify the notation once again, we introduce the following measure of difference in the degree of positionality between a potential mimicker (indicated by a hat) and the (mimicked) employed low-ability type:

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[15] The proofs of observations 1, 2, and 3 below follow as special cases of the proof of Proposition 1 in subsection 3.2.

$$\alpha^d = \frac{\lambda \hat{u}_c^1(\hat{\alpha}^2 - \alpha^1)}{\gamma N(1-\bar{\alpha})} = \frac{\lambda(\hat{v}_c^1 + \hat{v}_\Delta^1)(\hat{\alpha}^2 - \alpha^1)}{\gamma N(1-\bar{\alpha})}, \tag{12}$$

where $\alpha^d > 0$ if the potential mimicker is more positional than the low-ability type (i.e., if $\hat{\alpha}^2 > \alpha^1$), and vice versa. By using the expressions for τ^i, i=1,2, from Observation 1, we can then derive the following result:

Observation 2. (Aronsson and Johansson-Stenman, 2008) *If* $\alpha^j > 0$ *for j=1, 2, u, and* $N_e^1 \equiv N^1$*, the policy rules for marginal income taxation can be written as*

$$T'(w^i l^i) = \tau^i + (1-\tau^i)\bar{\alpha} - \frac{\alpha^d(1-\tau^i)(1-\bar{\alpha})}{1-\alpha^d} \quad \textit{for } i=1,2. \tag{13}$$

Observation 2 decomposes the policy rule into three basic incentives for marginal taxation. The first is the incentives that would be present also in the absence of any concerns for relative consumption, summarized by τ^i in equations (11). The second is the corrective motive for taxation, which depends on the average decree of positionality, $\bar{\alpha}$ (reflecting the marginal positional externality per unit of consumption). Note that this component is scaled by $1-\tau^i$ since the fraction of the marginal income tax away for other reasons does not give rise to any externalities. Finally, the policy maker can relax the self-selection constraint through a policy induced increase in $\bar{c}$ (by lowering the marginal tax rate) if the mimicker is more positional that the low-ability type ($\alpha^d > 0$), and through a policy induced decrease in $\bar{c}$ (by increasing the marginal tax rate) if the low-ability type is more positional than the mimicker ($\alpha^d < 0$).

Note that equation (13) reduces to read $T'(w^i l^i) = \tau^i + (1-\tau^i)\bar{\alpha}$ for i=1,2, if agents share the same leisure separable utility function (since this case means $\hat{\alpha}^2 = \alpha^1$ such that $\alpha^d = 0$). Finally, in a full information setting where individual ability is public information, equation (13) implies $T'(w^i l^i) = \bar{\alpha}$ for i=1,2 (since $\lambda = \tau^i = \alpha^d = 0$ in that case). Thus, if earnings-ability were publicly observable, it follows that lump-sum redistribution would be feasible, meaning that marginal income taxation would be used solely for purposes of externality correction.

Observation 3. *If* $\alpha^j \equiv 0$ *for j=1, 2, u, and* $N_e^1 < N^1$*, the policy rules for marginal income taxation are given by*

$$T'(w^1l^1) = \tau^1 + \frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1 - U^u}{\gamma w^1 l^1} \tag{14a}$$

$$T'(w^2l^2) = \tau^2 + \left(\frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1 - U^u}{\gamma w^1 l^1}\right)\phi \frac{F_{L^1L^2}}{F_{L^1L^1}}. \tag{14b}$$

Observation 3 shows that equilibrium unemployment among the low-skilled adds an additional motive for marginal taxation compared to the conventional policy rules in equations (11): to increase the number of employed persons of the low-ability type, which is captured by the second and third terms on the right hand side of equation (14a) and the analogous expression within brackets in equation (14b). The second term on the right hand side of equation (14a) is the net fiscal gain following if one additional individual becomes employed. If $T(w^1l^1)+b>0$ (which seems to be a plausible assumption), there is an incentive to increase the marginal income tax implemented for employed low-ability individuals in order to decrease their labor supply which, in turn, leads to an increase in the number of employed persons. Similarly, if we assume that $U^1 > U^u$, which rules out the possibility of voluntary unemployment, the third term on the right hand side is interpretable in terms of the welfare gain to the individual (measured per unit of income and relative to the marginal cost of public funds), of switching from unemployment to employment. This component also works to increase in the number of employed persons of the low-ability type through a tax induced decrease in the hours of work per employee. Finally, if we continue to assume that $T(w^1l^1)+b>0$ and $U^1 > U^u$, there is a corresponding incentive to tax the income of high-ability individuals at a lower marginal rate than in the conventional model, ceteris paribus, if the two skill-types are technical complements in production such that $F_{L^1L^2} > 0$.[16]

3.2 Main Results

With these preliminaries at our disposal, Proposition 1 presents the main results.

[16] Similar tax policy implications of equilibrium unemployment have been derived by Aronsson and Sjögren (2004) in a model with trade-union wage formation and Aronsson and Micheletto (forthcoming) under efficiency wage formation among the low-skilled.

Proposition 1. *If* $\alpha^j > 0$ *for j=1, 2, u, and* $N_e^1 < N^1$*, the marginal income taxes satisfy the following policy rules:*

$$T'(w^1l^1) = \tau^1 + \frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1 - U^u}{\gamma w^1 l^1}\frac{1-\bar{\alpha}}{1-\alpha^d} - \tau^1 \frac{\bar{\alpha}-\alpha^d}{1-\alpha^d} \qquad (15a)$$

$$T'(w^2l^2) = \tau^2 + (1-\tau^2)\bar{\alpha} - \frac{\alpha^d(1-\tau^2)(1-\bar{\alpha})}{1-\alpha^d} + \left(\frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1-U^u}{\gamma w^1l^1}\frac{1-\bar{\alpha}}{1-\alpha^d} - \frac{\bar{\alpha}-\alpha^d}{1-\alpha^d}\right)\phi\frac{F_{L^1L^2}}{F_{L^1L^1}}. \qquad (15b)$$

Proof: See the Appendix.

Consider first the marginal income tax implemented for the low-ability type in equation (15a). To facilitate the interpretation, we start with the special case where earnings-ability is public information, in which the self-selection constraint is redundant. This special case means that $\lambda = 0$ and, as a consequence, $\tau^1 = \alpha^d = 0$, and equation (15a) reduces to read

$$T'(w^1l^1) = \frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1-U^u}{\gamma w^1l^1}\left(1-\bar{\alpha}\right). \qquad (16)$$

Equation (16) is reminiscent of equation (14a) in the special case where $\tau^1 = 0$, except that the final term is multiplied by $1-\bar{\alpha}$. By comparison, if the labor market were competitive, we showed above that the analogue to equation (16) would take the form $T'(w^1l^1) = \bar{\alpha}$ (a special case of equation [13]), which is interpretable as a Pigouvian tax targeting the positional externality that each individual imposes on other people.

To understand equation (16), two things are particularly important. First, equation (16) contains no externality-correcting component. This is because, under involuntary unemployment, a policy induced change in the hours of work per employed low-ability individual affects the positional externality via two offsetting channels: there is an incentive to (i) increase the marginal tax rate in order to reduce $\bar{c}$ via a lower c^1, and (ii) decrease the marginal tax rate in order to reduce $\bar{c}$ through a lower N_e^1 in which case fewer persons can contribute to the positional externality through their hours of work.[17] The latter is accomplished by increasing the hours of work per employed low-ability individual. These two effects cancel out in the

[17] Recall that the unemployed make no active choices in the model, meaning that their consumption does not depend directly on the marginal tax policy.

special case where individual earnings-ability is observable.[18] In a competitive labor market, on the other hand, only the first of these policy incentives is present, which case the positional externality leads to increased marginal taxation (and possibly a high marginal tax rate considering the empirical estimates of $\bar{\alpha}$ discussed above).

Second, the scale factor $1-\bar{\alpha}$ appears in equation (16) due to a discrepancy between the utility gain of increased employment to the individual and the value society attaches to this utility gain. Since relative consumption leads to a welfare cost (through the positional externality), the government attaches less value to this gain in individual well-being (and thus increases the marginal income tax rate less for this particular reason) than it would have done if the individuals were completely non-positional. Therefore, whereas relative consumption concerns motivate a higher marginal income tax rate (allowing the policy maker to correct for the positional externality) if the labor market is competitive, such concerns motivate a lower (!) marginal income tax rate (to adjust for a discrepancy between the private and social benefits of increased employment) if the labor market is characterized by involuntary unemployment.

Consider next the less restrictive (and frequently discussed) special case where earnings-ability is private information (such that $\lambda > 0$), while leisure is weakly in terms of the utility function in the sense that $v^i(c^i, z^i, c^i - \bar{c}) = h^i(g(c^i, c^i - \bar{c}), z^i)$, in turn implying $\alpha^d = 0$. This case is common in numerical studies on optimal taxation when consumers are concerned about their relative consumption (e.g., Kanbur and Tuomala, 2013; Aronsson and Johansson-Stenman, 2018). As indicated above, if the labor market were competitive this would result in the following policy rule for marginal taxation: $T'(w^i l^i) = \tau^i + (1-\tau^i)\bar{\alpha}$ for i=1,2 (which is a special case of equation [13a]). Under unemployment, the corresponding policy rule follows from equation (15a), i.e.,

[18] If the number of workers and the hours supplied per worker are not perfect substitutes (e.g., due to a hiring cost), the two offsetting effects described above are still present even if they do not cancel out completely. This case means that equation (16) changes to read

$$T'(w^1 l^1) = \bar{\alpha}\left(1 + \frac{\partial N_e^1}{\partial l^1}\frac{l^1}{N_e^1}\right) - \frac{T(w^1 l^1) + b}{w^1 l^1}\frac{\partial N_e^1}{\partial l^1}\frac{l^1}{N_e^1} - \frac{U^1 - U^u}{\gamma w^1 l^1}\frac{\partial N_e^1}{\partial l^1}\frac{l^1}{N_e^1}\left(1-\bar{\alpha}\right).$$

The corrective tax element is given by the first term on the right hand side, which falls short of the average degree of positionality, $\bar{\alpha}$. If the number of workers and the hours supplied per worker are perfect substitutes, we have $\partial N_e^1 / \partial l^1 = -N_e^1 / l^1$, which explains why the corrective element vanishes completely in equation (16).

$$T'(w^1l^1) = \tau^1 + \frac{T(w^1l^1)+b}{w^1l^1} + \frac{U^1-U^u}{\gamma w^1l^1}\left(1-\bar{\alpha}\right) - \tau^1\bar{\alpha}\,. \tag{17}$$

Equation (17) differs from equation (16) by the final term on the right hand side, which is interpretable as a corrective tax component: it reflects the net welfare effect of an increase in the positional externality through a policy induced change in the hours of work per employed low-ability individual. The intuition is again that the positional externality gives rise to two counteracting policy incentives when the labor market is characterized by unemployment. In this case, however, where earnings-ability is private information, the incentive to decrease the marginal income tax in order to reduce $\bar{c}$ via a lower N_e^1 dominates the incentive to increase the marginal income tax in order to reduce $\bar{c}$ via a lower c^1. This is because the latter effect is scaled down by $1-\tau^1<1$ if earnings-ability is private information (as seen from the second term on the right hand side of equation [13]), while it would not be scaled down if earnings-ability were public information (in which case $\tau^1=0$). Therefore, the incentive to decrease the marginal income tax rate for the low-ability type in response to relative consumption concerns described in the context of equation (16) is further strengthened by externality correction (!).

Returning to the general case illustrated in equation (15a), we can see that the qualitative effect of the positional externality now depends on the difference $\bar{\alpha}-\alpha^d$ instead of on $\bar{\alpha}$ alone as it did in the special case of leisure separability. With a general (non-separable) utility function, the welfare gain of a decrease in $\bar{c}$ does not only depend on the sum of the marginal willingness to pay to avoid the externality (as reflected in the average degree of positionality, $\bar{\alpha}$); it also depends on whether potential mimickers are willing to pay more ($\alpha^d>0$) or less ($\alpha^d<0$) at the margin than the mimicked agents. Consequently, a policy maker would exploit this discrepancy in marginal willingness to pay in order to relax the self-selection constraint. If the low-ability type is willing to pay more, there is an unambiguous welfare gain of decreasing $\bar{c}$ (as $\bar{\alpha}-\alpha^d$ is always positive in that case), meaning that the interpretation presented above for the leisure separable utility function carries over to the general model. However, if the mimicker is willing to pay more than the low-ability type, the qualitative policy response to relative concerns will depend on whether the incentive for externality correction (reflected in $\bar{\alpha}$) dominates, or is dominated by, the incentive to relax the self-selection constraint via a policy induced adjustment in $\bar{c}$.

Finally, turning to the marginal tax policy implemented for the high-ability type in equation (15b), the first line coincides with the policy rule in a competitive labor market, with the same interpretation as equation (13), while the second line is the “add on” caused by involuntary unemployment. The second line is thus interpretable in the same general way as equation (14b), albeit with one important exception: the “add on” now contains an additional component, since an increase in the number of employed low-ability individuals - caused by a tax induced increase in the labor supply among high-ability individuals - leads to an increase in the positional externality (the third term in brackets). The latter is a net welfare cost (benefit) if $\bar{\alpha} > (<)\ \alpha^d$. If $\bar{\alpha} > \alpha^d$, which appears to us as a plausible assumption, there are two distinct motives for taxing the income of high-ability individuals in response to positional externalities: first, to reduce the direct positional externality caused by each such individual’s consumption and, second, to counteract the indirect contribution to the positional externality following when a smaller l^2 leads to a decrease in N_e^1. As such, by comparing equations (13), (14b), and (15b), we can see that the positional externality motivates a larger increase in the marginal income tax implemented for high-ability individuals under involuntary unemployment than it would do under competition in the labor market, ceteris paribus.

4. Summary

As far as we know, the present paper is the first to analyze the problem of optimal taxation in a model characterized both by positional externalities and by involuntary unemployment. The implications of positional externalities for marginal income taxation are shown to depend on the functioning of the labor market. We have considered a model-economy where a minimum wage-policy leads to involuntary equilibrium unemployment among the low-skilled. Under equilibrium unemployment, the optimal tax policy responses to positional externalities differ in a fundamental way from those derived in models with competitive labor markets (all earlier studies on optimal taxation and positional externalities that we are aware of). We have shown that (i) relative consumption concerns are likely to imply a lower (not higher) marginal income tax facing low-ability individuals, and (ii) the marginal income tax implemented for high-ability individuals should increase more in response to positional externalities under involuntary unemployment than under competition in the labor market.

We have chosen a highly stylized model in order to make the results as clear as possible, but the qualitative insights are generalizable to a much broader set of assumptions regarding wage-formation, the production function, and the number of productivity types. What is crucial is that there is some degree of substitutability between the labor supply per worker and the number of workers in equilibrium. Overall, we conclude that the assumption of competition in the labor market, when analyzing the tax policy implications of relative consumption concerns, is not innocuous.

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Appendix

Proof of Proposition 1

The Lagrangean corresponding to the social decision-problem is given by

$$
\begin{aligned}
\mathcal{L} &= N_e^1 v^1(c^1, z^1, c^1 - \bar{c}) + N^2 v^2(c^2, z^2, c^2 - \bar{c}) + \left(N^1 - N_e^1\right) v(b, 1, b - \bar{c}) \\
&+ \lambda\left(v^2(c^2, z^2, c^2 - \bar{c}) - \hat{v}^2(c^1, 1 - \phi l^1, c^1 - \bar{c})\right) \\
&+ \gamma\left(F(N_e^1 l^1, N^2 l^2) - N_e^1 c^1 - (N^1 - N_e^1) b - N^2 c^2\right)
\end{aligned}
$$

The social first-order conditions consumption for c^1, l^1, c^2, l^2, and b can be written as

$$\frac{\partial \mathcal{L}}{\partial c^1} = N_e^1\left(v_c^1 + v_\Delta^1\right) - \lambda\left(\hat{v}_c^2 + \hat{v}_\Delta^2\right) - \gamma N_e^1 + \frac{\partial \mathcal{L}}{\partial \bar{c}} \frac{N_e^1}{N} = 0 \tag{A1}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial l^1} &= -N_e^1 v_z^1 + \lambda \hat{v}_z^2\left(\phi + l^1 \frac{\partial \phi}{\partial l^1}\right) + \gamma\left(w^1 N_e^1 + \left(w^1 l^1 - c^e + b\right)\frac{\partial N_e^1}{\partial l^1}\right) \\
&+ \frac{\partial \mathcal{L}}{\partial \bar{c}} \frac{\partial \bar{c}}{\partial l^1} + \left(v^1 - v^u\right)\frac{\partial N_e^1}{\partial l^1} = 0
\end{aligned} \tag{A2}
$$

$$\frac{\partial \mathcal{L}}{\partial c^2} = \left(N^2 + \lambda\right)\left(v_c^2 + v_\Delta^2\right) - \gamma N^2 + \frac{\partial \mathcal{L}}{\partial \bar{c}} \frac{N^2}{N} = 0 \tag{A3}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial l^2} &= -\left(N^2 + \lambda\right) v_z^2 + \lambda \hat{v}_z^2 l^1 \frac{\partial \phi}{\partial l^2} + \gamma\left(w^2 N^2 + \left(w^1 l^1 - c^e + b\right)\frac{\partial N_e^1}{\partial l^2}\right) \\
&+ \frac{\partial \mathcal{L}}{\partial \bar{c}} \frac{\partial \bar{c}}{\partial l^2} + \left(v^1 - v^u\right)\frac{\partial N_e^1}{\partial l^2} = 0
\end{aligned} \tag{A4}
$$

$$\frac{\partial \mathcal{L}}{\partial b} = \left(N^1 - N_e^1\right)\left(v_c^u + v_\Delta^u\right) - \gamma\left(N^1 - N_e^1\right) + \frac{\partial \mathcal{L}}{\partial \bar{c}} \frac{N^1 - N_e^1}{N} = 0\,. \tag{A5}$$

Note that

$$\frac{\partial \mathcal{L}}{\partial \overline{c}} = -N_e^1 v_\Delta^1 - \left(N^2 + \lambda\right) v_\Delta^2 - \left(N^1 - N_e^1\right) v_\Delta^u + \lambda \hat{v}_\Delta^2 \\ = -(v_c^1 + v_\Delta^1)\alpha^1 - (\delta^2 + \lambda)(v_c^2 + v_\Delta^2)\alpha^2 - \delta^u (v_c^u + v_\Delta^u)\alpha^u + \lambda(\hat{v}_c^2 + \hat{v}_\Delta^2)\hat{\alpha}^2 . \quad \text{(A6)}$$

By solving equations (A1), (A3), and (A5) for $N_e^1(v_c^1 + v_\Delta^1)$, $(N^2 + \lambda)(v_c^2 + v_\Delta^2)$, and $(N^1 - N_e^1)(v_c^u + v_\Delta^u)$, respectively, and substituting into equation (A6), we obtain

$$\frac{\partial \mathcal{L}}{\partial \overline{c}} = -\left(\lambda(\hat{v}_c^2 + \hat{v}_\Delta^2) + \gamma N_e^1 - \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{N_e^1}{N} \right)\alpha^1 + \left(\gamma N^2 - \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{N^2}{N} \right)\alpha^2 \\ - \left(\gamma \left(N^1 - N_e^1\right) - \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{N^1 - N_e^1}{N} \right)\alpha^u + \lambda(\hat{v}_c^2 + \hat{v}_\Delta^2)\hat{\alpha}^2 . \quad \text{(A7)}$$

Collecting terms and using the definition of $\overline{\alpha}$ in equation (7) give

$$\frac{\partial \mathcal{L}}{\partial \overline{c}} = -\gamma N \frac{\overline{\alpha}}{1-\overline{\alpha}} + \frac{\lambda(\hat{v}_c^1 + \hat{v}_\Delta^1)(\hat{\alpha}^2 - \alpha^1)}{(1-\overline{\alpha})} = \gamma N \frac{-\overline{\alpha} + \alpha^d}{1-\overline{\alpha}} . \quad \text{(A8)}$$

To derive the marginal income tax rate implemented for the employed low-ability type, combine equations (A1) and (A2) to derive

$$\frac{v_z^1}{v_c^1 + v_\Delta^1}\left((\hat{v}_c^2 + \hat{v}_\Delta^2) + \gamma N_e^1 - \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{N_e^1}{N} \right) = \lambda \hat{v}_z^2 \phi + \gamma \left(w^1 N_e^1 + \left(T(w^1 l^1) + b\right)\frac{\partial N_e^1}{\partial l^1} \right) \\ + \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{\partial \overline{c}}{\partial l^1} + \left(v^1 - v^u\right)\frac{\partial N_e^1}{\partial l^1}$$

where we have used $T(w^1 l^1) = w^1 l^1 - c^1$. Solving for $w^1 - v_z^1 / (v_c^1 + v_\Delta^1)$, rearranging, and using $w^1 T'(w^1 l^1) = w^1 - v_z^1 / (v_c^1 + v_\Delta^1)$ from the private first-order condition, we obtain

$$\gamma N_e^1 w^1 T'(w^1 l^1) = \lambda(\hat{v}_c^2 + \hat{v}_\Delta^2)\left(\frac{v_z^1}{v_c^1 + v_\Delta^1} - \phi \frac{\hat{v}_z^2}{\hat{v}_c^2 + \hat{v}_\Delta^2} \right) - \gamma \left(T(w^1 l^1) + b\right)\frac{\partial N_e^1}{\partial l^1} \\ - \frac{v_z^1}{v_c^1 + v_\Delta^1} \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{N_e^1}{N} - \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{\partial \overline{c}}{\partial l^1} - \left(v^1 - v^u\right)\frac{\partial N_e^1}{\partial l^1} , \quad \text{(A9)}$$

which can also be written to read, if we use the definition of τ^1 in equation (11a),

$$T'(w^1 l^1) = \tau^1 - \frac{1}{w^1 N_e^1}\left(T(w^1 l^1) + b\right)\frac{\partial N_e^1}{\partial l^1} - \frac{1}{\gamma w^1 N} \frac{v_z^1}{v_c^1 + v_\Delta^1} \frac{\partial \mathcal{L}}{\partial \overline{c}} \\ - \frac{1}{\gamma w^1 N_e^1} \frac{\partial \mathcal{L}}{\partial \overline{c}} \frac{\partial \overline{c}}{\partial l^1} - \frac{v^1 - v^u}{\gamma w^1 N_e^1} \frac{\partial N_e^1}{\partial l^1} . \quad \text{(A10)}$$

Now, use $\partial\overline{c}/\partial l^1 = (c^1 - b)(\partial N_e^1/\partial l^1)/N$ and $\left(v_z^1/(v_c^1+v_\Delta^1)\right)/w^1 = 1 - T'(w^1 l^1)$ together with equations (2a) and (A8) to rewrite equation (A10) as follows:

$$
\begin{aligned}
T'(w^1l^1) &= \tau^1 + \frac{T(w^1l^1)+b}{w^1l^1} + \left(1-T'(w^1l^1)\right)\frac{\overline{\alpha}-\alpha^d}{1-\overline{\alpha}} \\
&\quad - \frac{\overline{\alpha}-\alpha^d}{1-\overline{\alpha}}\frac{w^1l^1 - T(w^1l^1)-b}{w^1l^1} + \frac{v^1-v^u}{\gamma w^1 l^1}
\end{aligned}
. \tag{A11}
$$

Collecting $T'(w^1l^1)-$terms and rearranging gives

$$
\begin{aligned}
T'(w^1l^1) &= \tau^1 \frac{1-\overline{\alpha}}{1-\alpha^d} + \frac{T(w^1l^1)+b}{w^1l^1}\frac{1-\overline{\alpha}}{1-\alpha^d} + \frac{\overline{\alpha}-\alpha^d}{1-\alpha^d} \\
&\quad - \frac{\overline{\alpha}-\alpha^d}{1-\alpha^d}\frac{w^1l^1 - T(w^1l^1)-b}{w^1l^1} + \frac{v^1-v^u}{\gamma w^1 l^1}\frac{1-\overline{\alpha}}{1-\alpha^d}
\end{aligned}
. \tag{A12}
$$

Add and subtract $\tau^1 + (1-\tau^1)\overline{\alpha}$ from the right hand side of equation (A12). Rearrangement gives

$$
\begin{aligned}
T'(w^1l^1) &= \tau^1 + (1-\tau^1)\overline{\alpha} - \frac{\alpha^d(1-\tau^1)(1-\overline{\alpha})}{1-\alpha^d} + \frac{T(w^1l^1)+b}{w^1l^1}\frac{1-\overline{\alpha}}{1-\alpha^d} \\
&\quad - \frac{\overline{\alpha}-\alpha^d}{1-\alpha^d}\frac{w^1l^1 - T(w^1l^1)-b}{w^1l^1} + \frac{v^1-v^u}{\gamma w^1 l^1}\frac{1-\overline{\alpha}}{1-\alpha^d}
\end{aligned}
\tag{A13}
$$

or

$$
T'(w^1l^1) = \tau^1 + \frac{T(w^1l^1)+b}{w^1l^1} - \tau^1\frac{\overline{\alpha}-\alpha^d}{1-\alpha^d} + \frac{v^1-v^u}{\gamma w^1 l^1}\frac{1-\overline{\alpha}}{1-\alpha^d} \tag{A14}
$$

which coincides with equation (15a). Equation (11a) can be derived from the special case of equation (A10) where $\partial\mathcal{L}/\partial\overline{c} = \partial N_e^1/\partial l^1 = 0$; equation (13) from the special case of equation (A10) where $\partial N_e^1/\partial l^1 = 0$; and equation (14a) from the special case of equation (A10) where $\partial\mathcal{L}/\partial\overline{c} = 0$.

Turning to the high-ability type, we can use equations (A3) and (A4) together with the private first-order condition to derive

$$
\begin{aligned}
T'(w^2l^2) &= \tau^2 - \frac{1}{w^2N^2}\left(T(w^1l^1)+b\right)\frac{\partial N_e^1}{\partial l^2} - \frac{1}{\gamma w^2 N}\frac{v_z^2}{v_c^2+v_\Delta^2}\frac{\partial\mathcal{L}}{\partial\overline{c}} \\
&\quad - \frac{1}{\gamma w^2 N^2}\frac{\partial\mathcal{L}}{\partial\overline{c}}\frac{\partial\overline{c}}{\partial l^2} - \frac{v^1-v^u}{\gamma w^2 N^2}\frac{\partial N_e^1}{\partial l^2}
\end{aligned}
\tag{A15}
$$

which is analogous to equation (A10). Now, use $\partial\bar{c}/\partial l^2 = (c^1 - b)(\partial N_e^1/\partial l^2)/N$ and $\left(v_z^2/(v_c^2 + v_\Delta^2\right)/w^2 = 1 - T'(w^2 l^2)$ together with equation (2b) and (A8) to rewrite equation (A15) as follows:

$$T'(w^2l^2) = \tau^2 + \frac{T(w^1l^1)+b}{w^2l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}} + \left(1 - T'(w^2l^2)\right)\frac{\bar{\alpha}-\alpha^d}{1-\bar{\alpha}} - \frac{\bar{\alpha}-\alpha^d}{1-\bar{\alpha}}\frac{w^1l^1 - T(w^1l^1) - b}{w^2l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}} + \frac{v^1 - v^u}{\gamma w^2 l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}}. \tag{A16}$$

Collecting $T'(w^2l^2)-$ terms and rearranging gives

$$T'(w^2l^2) = \tau^2\frac{1-\bar{\alpha}}{1-\alpha^d} + \frac{T(w^1l^1)+b}{w^2l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}}\frac{1-\bar{\alpha}}{1-\alpha^d} + \frac{\bar{\alpha}-\alpha^d}{1-\alpha^d} - \frac{\bar{\alpha}-\alpha^d}{1-\alpha^d}\frac{w^1l^1 - T(w^1l^1) - b}{w^2l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}} + \frac{v^1 - v^u}{\gamma w^2 l^1}\frac{F_{L^1L^2}}{F_{L^1L^1}}\frac{1-\bar{\alpha}}{1-\alpha^d}. \tag{A17}$$

Rearrangements give equation (15b). Equation (11b) can be derived from the special case of equation (A15) where $\partial\mathcal{L}/\partial\bar{c} = \partial N_e^1/\partial l^2 = 0$; equation (13) from the special case of equation (A15) where $\partial N_e^1/\partial l^2 = 0$; and equation (14b) from the special case of equation (A15) where $\partial\mathcal{L}/\partial\bar{c} = 0$.■

The Minimum Wage

If there is a minimum wage such that $w^1 = \bar{w}^1$ and $N_e^1 < N^1$, and if the minimum wage is decided on by the government, the social first-order condition for this minimum wage can be written as follows (under an interior solution):

$$\frac{\partial\mathcal{L}}{\partial\bar{w}^1} = \lambda\hat{v}_z^2 l^1\frac{\partial\phi}{\partial\bar{w}^1} + \left(v^1 - v^u\right)\frac{\partial N_e^1}{\partial\bar{w}^1} + \gamma\left(T(w^1l^1)+b\right)\frac{\partial N_e^1}{\partial\bar{w}^1} + \frac{\partial\mathcal{L}}{\partial\bar{c}}\frac{\partial\bar{c}}{\partial\bar{w}^1} = 0 \tag{A18}$$

where

$$\frac{\partial\phi}{\partial\bar{w}^1} = \frac{\partial\left(\bar{w}^1/w^2(l^1,l^2,\bar{w}^1)\right)}{\partial\bar{w}^1} = \frac{w^2 - \bar{w}^1(\partial w^2/\partial\bar{w}^1)}{(w^2)^2} > 0$$

$$\frac{\partial\bar{c}}{\partial\bar{w}^1} = \frac{c^1 - c^u}{N}\frac{\partial N_e^1}{\partial\bar{w}^1}$$

and $\partial N_e^1/\partial\bar{w}^1 < 0$.